On the geometry of the Humbert surface of square discriminant

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Isogenies of elliptic curves over \mathbb{Q}

<u>Question</u>: Which integers $m \ge 2$ can be the degree of a (cyclic) isogeny $\phi \colon E \to E'$ where E, E', and ϕ are defined over \mathbb{Q} ?

curves defined over \mathbb{Q} , then the degree of ϕ is ≤ 19 , or ∈ {21,25,27,37,43,67,163}.

<u>Theorem (Mazur, Kenku)</u>: If $\phi: E \to E'$ is a (cyclic) isogeny between elliptic



Upgrade to genus 2

<u>Question</u>: For which integers $N \ge 2$ do there there exist some genus 2 curve C/\mathbb{Q} an elliptic curve E/\mathbb{Q} and a morphism

over \mathbb{Q} and "minimal" degree N?

 ψ does not factor through an isogenv

Example: Take $E : y^2 = f(x)$ and $C : y^2 = f(x^2)$ so that we have $C \to E$ induced by $x \mapsto x^2$ with N = 2.

 $\psi \colon C \to E$



In terms of Galois representations

different minimal degree N morphism $\psi': C \to E'$.



The existence of $E \times E' \rightarrow Jac(C)$ implies there exists an isomorphism

of $Gal(\mathbb{Q}/\mathbb{Q})$ -modules.

The converse also holds so long as E and E' are not isogenous (and the isomorphism) $E[N] \cong_{\mathbb{O}} E'[N]$ is antisymplectic wrt the Weil pairing).

The morphism $\psi \colon C \to E$ induces an isogeny $E \times E' \to \text{Jac}(C)$ over \mathbb{Q} and a





Ensures that the image of $E \times E' \rightarrow \operatorname{Jac}(C)$ is a Jacobian



Frey – Mazur conjecture

geometrically isogenous.

<u>Conjecture (Fisher)</u>: If one restricts to N is prime, one can take $N_0 = 19$ in the Frey—Mazur conjecture.

<u>Conjecture</u> (Frey – Mazur): There exists an integer $N_0 > 0$ such that for all $N \ge N_0$ any pair of elliptic curves E/\mathbb{Q} and E'/\mathbb{Q} with $E[N] \cong_{\mathbb{Q}} E'[N]$ are



Frey—Mazur conjecture

geometrically isogenous.

<u>Theorem (Frey)</u>: If the Frey—Mazur conjecture then the asymptotic Fermat conjecture holds.

<u>Conjecture</u> (Frey – Mazur): There exists an integer $N_0 > 0$ such that for all $N \ge N_0$ any pair of elliptic curves E/\mathbb{Q} and E'/\mathbb{Q} with $E[N] \cong_{\mathbb{Q}} E'[N]$ are



Example / Theorem (F.): There exists a genus 2 curve C/\mathbb{Q} with minimal degree 15 morphism over \mathbb{Q} to

 $E: y^2 + xy + y = x^3 - x^2 - 5978298027424617040871x - 177915816685044386506178867920438$

Which are not (geometrically) isogenous and have conductor

Also:

- ∞ many for $N \leq 11$ (Kumar + ...)
- ∞ many for N = 12 (F.)
- ∞ many for N = 13 (Fisher)

• ~
$$10 \text{ for } N = 14$$
 (F.)

- One for N = 15 (F.)
- One for N = 17 (Fisher)

- $E': y^2 = x^3 2135607437331989841943540710782811x + 37915783123298007085317147066745283477127543370806$
 - $3^3 \cdot 7 \cdot 11 \cdot 13 \cdot 23 \cdot 61 \cdot 199^2 \cdot 2341^2 \approx 10^{19}$



The relevant moduli spaces <u>Isogeny $\phi: E \to E'$ </u> <u>Coverings $\psi: C \to E$ </u> + data of *m*-isogeny $\phi \colon E \to E'$ Modular curve $X_0(m)$ Hilbert modular surface $Y_{(N^2)}$ such that $X_0(m)$

Mazur—Kenku "just" find the rational points on $X_0(m)$ for every $m \ge 2$.

{k-points on $Y_{(N^2)}$ } \leftrightarrow { ψ : $C \rightarrow E$ over k}

We "just" need to find the rational points on $Y_{(N^2)}$ for every $N \ge 2$.

exist a family of $\Psi: \mathscr{C} \to \mathscr{C}$ over $\mathbb{C}(s, t)$ (or $\mathbb{Q}(s, t)$)?

(birational to a) surface which is

- Rational if $N \leq 5$
- Elliptic K3 if N = 6,7
- Properly elliptic if N = 8,9,10
- General type if $N \ge 11$.

Question: Too hard over \mathbb{Q} , make it easier. For which integers N does there

Theorem (Hermann, Kani – Schanz): The Hilbert modular surface $Y_{-}(N^{2})$ is



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(symplectic isom.) Over $\mathbb{C}(t)$ are isogenous when $p \ge p_0$.

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Thm:
$$\kappa = \min(2, p_g - 1)$$

Theorem (Bakker-Tismerman): There exists p_0 so that any $E[p] \cong_{\mathbb{C}(t)} E'[p]$





Humbert surfaces

 $Y_{(N^2)}$ is general type? $Y_{(N^2)} \xrightarrow{2:1} \mathscr{H}_{(N^2)}$ $[\psi\colon C\to E]\mapsto [C]$

Remembers only that *C* admits a map to an elliptic curve (equivalently, unordered pairs $C \to E$ and $C \to E'$)

<u>But</u>: We had an example with N = 15, 17? Should this be unexpected since





Humbert surfaces

But: We had an example with N = 1: $Y_{(N^2)}$ is general type?

Probably not many points (Bombieri– Lang conjecture)

<u>But</u>: We had an example with N = 15, 17? Should this be unexpected since

$Y_{-}(N^{2}) \xrightarrow{2:1} \mathscr{H}_{-}(N^{2})$ $[\psi: C \to E] \mapsto [C]$ Lots of points N = 15,17

Humbert surfaces

Another reason: The surface $\mathscr{H}_{-}(N^2)$ is very natural from the point of view of moduli. We have a natural way of viewing



The union

 $\mathcal{H}_{(N^2)} \subset \mathcal{M}_2$

As defined it is only birational to its image

 $\mathcal{H}_{(N^2)}$

are the points in M_2 with Jacobians isogenous to a product of elliptic curves.

As principally polarised abelian varieties

N = p

is (birational to):

- A rational surface if $N \leq 16$ or if N = 18, 20, or 24,
- An elliptic K3 surface if N = 17,
- A properly elliptic surface if N = 19 or 21, and
- A surface of general type if $N \ge 22$ and $N \ne 24$.

 - Kumar ($N \le 11$) + ...
 - Fisher (N = 13) F. (N = 12,14,15,18,20,24)
 - There exists $\mathscr{C}/\mathbb{C}(u, v)$ which admits a

minimal cover of degree N to an elliptic curve (after base change to a quadratic extension)

Theorem (Hermann, F.): The Humbert surface of square discriminant $\mathscr{H}_{-}(N^{2})$

<u>Thm:</u> $\kappa = \min(2, p_g - 1)$



How?

Use that $\mathscr{H}_{-}(N^2)$ is a quotient of $Y_{-}(N^2)$ by an involution.

- 1. Understand fixed point locus on $Y_{-}(N^2)$ (genera, singularities, etc)
 - e.g., $X_{ns}^+(p^k)$ or $X_s^+(p^k)$ and N odd you get "extended Cartan"
- 2. Desingularise quotient of $Y_{(N^2)}$
 - blow-down Hecke correspondences (when N even, fancier X_H)
- 3. Understand (self)-intersections and singularities of Hecke correspondences on quotient to find elliptic fibres etc.



<u>Challenge</u>: Below is $j(\mathscr{E})j(\mathscr{E}') \in \mathbb{Q}(u,v)$ (also have $j(\mathscr{E}) + j(\mathscr{E}')$) for a pair of elliptic curves for which there exists a genus 2 curve $\mathscr{C}/\mathbb{Q}(u, v)$ with degree N = 24 maps to elliptic curves (over a quadratic extension of $\mathbb{Q}(u, v)$). Find \mathscr{C} .

232*u^32*v^15 + 508*u^32*v^14 + 1032*u^32*v^13 + 7814*u^32*v^12 + 17480*u^32*v' 1^31*v^17 + 18360*u^31*v^16 + 28520*u^31*v^15 + 133048*u^31*v^14 + 191544*u^31*v^13 + 413672*u^31*v^12 + 439880*u^31*v^11 + 365256*u^31*v^10 - 214488*u^31*v^9 - 1072000*u^31*v^8 - 1950672*u^31*v^1 77568*u^31*v - 8192*u^31 - 864*u^30*v^20 + 13568*u^30*v^19 + 145040*u^30*v^18 + 18296*u^30*v^16 - 395680*u^30*v^14 - 3683512*u^30*v^13 - 8000276*u^30*v^12 - 15274096*u^30*v^11 - 19702684*u^30*v^10 -10215408*u^30*v^6 + 11509952*u^30*v^5 + 8428800*u^30*v^4 + 4376320*u^30*v^2 + 307200*u^30*v + 25344*u^29*v^22 + 89888*u^29*v^21 + 263584*u^29*v^20 - 1400256*u^29*v^19 + 32200*u^29*v^18 - $-32281664 * u^{29} * v^{13} + 18100432 * u^{29} * v^{12} + 109044696 * u^{29} * v^{11} + 194109248 * u^{29} * v^{10} + 217391472 * u^{29} * v^{9} + 164065440 * u^{29} * v^{6} - 20910336 * u^{29} * v^{5} - 20259456 * u^{29} * v^{12} + 109044696 * u^{29} * v^{2} - 541184 * u^{29} * v^{6} - 3100432 * u^{29} * v^{11} + 109044696 * u^{29} * v^{11} + 10904669 * u^{29} * v^{29} * v^{11} + 10904669 * u^{29} * v^{29} * v^{29$ $212000 * u^{28*v^{23} - 482976 * u^{28*v^{22} - 4806464 * u^{28*v^{21} + 8583832 * u^{28*v^{20} - 9140248 * u^{28*v^{19} + 5865124 * u^{28*v^{16} + 215968240 * u^{28*v^{15} + 618995064 * u^{28*v^{14} + 895190560 * u^{28*v^{13} + 695926922 * u^{28*v^{12} + 17161200 * u^{28*v^{11}}}}$ 893488*u^28*v^9 - 904578276*u^28*v^8 - 497370336*u^28*v^7 - 174591712*u^28*v^6 - 24186592*u^28*v^5 + 12576160*u^28*v^2 + 313344*u^28*v + 18688*u^28 + 26112*u^27*v^26 + 201152*u^27*v^25 - 2609536*u^27*v^24 - 3213760*u^27*v^23 + 2368128*v^2 + 313344*u^28*v^2 + 3188*u^28*v^2 + 3188*v^28*v^2 + 3188*v^2 $1117351256*u^{26*v^{21}} + 3753193420*u^{26*v^{20}} + 2827547200*u^{26*v^{19}} - 4849258960*u^{26*v^{18}} - 18428217792*u^{26*v^{15}} + 8962517540*u^{26*v^{14}} + 32778836944*u^{26*v^{13}} + 38081897528*u^{26*v^{12}} + 2472232992$ 26*v^9 - 4163548360*u^26*v^8 - 2245872416*u^26*v^7 - 721954480*u^26*v^6 - 158600832*u^26*v^5 - 36059776*u^26*v^3 - 2786688*u^26*v^2 - 405504*u^26*v - 25344*u^26 + 27648*u^25*v^30 - 452736*u^25*v^29 - 3059712*u^25*v^28 + 27102016*u^25*v^2 - 22310071216*u^25*v^2 - 48804466464*u^25*v^1 + 1112672624*u^25*v^18 + 114550613488*u^25*v^17 + 182825653456*u^25*v^16 + 149261780792*u^25*v^15 + 31806406464*u^25*v^2 - 488044664664*u^25*v^2 - 4880446646664*u^25*v^2 - 48804466466664*u^25*v^2 - 48804466466664*u^25*v^2 - 4880446666664*u^25*v^2 - 488044666664*u^25*v^2 - 4880446666664*u^25*v^2 - 4880446666664*u^25*v^2 - 488044666664*u^25*v^2 - 4880446666664*u^25*v^2 - 4880446666664*u^25*v^2 - 488044666664*u^25*v^2 - 4880446666664*u^25*v^2 - 488044666664*u^25*v^2 - 48804*v^2 - 4880*v^2 - 4880*v^2 13 - 116522570720*u^25*v^12 - 81712567088*u^25*v^11 - 30907115904*u^25*v^10 - 1967667504*u^25*v^9 + 4511311584*u^25*v^8 + 2565272688*u^25*v^7 + 644190720*u^25*v^6 + 50694144*u^25*v^5 L + 1036224*u^24*v^30 + 32330880*u^24*v^29 - 69487936*u^24*v^28+ 191940576*u^24*v^27 + 2044736160*u^24*v^26 - 6682128000*u^24*v^25 - 2409 '24*v^18 - 410634394520*u^24*v^17 - 779262138811*u^24*v^16 - 710580427200*u^24*v^15 - 310776184544*u^24*v^14 + 78375947616*u^24*v^13 + 229484460272*u^24*v^12 + 177940268672*u^24*v^11 + 76173697904*u^24*v^10 + 1538274 4*v^5 + 17686464*u^24*v^4 + 8909952*u^24*v^3 + 1773504*u^24*v^2 + 193536*u^24*v + 9216*u^24 - 202752*u^23*v^33 + 3741696*u^23*v^32 + 14515200*u^23*v^31 - 120684288*u^23*v^30 + 144292224*u^23*v^29 - 145515200*u^23*v^32 + 14515200*u^23*v^31 - 120684288*u^23*v^30 + 144292224*u^23*v^29 - 145515200*u^23*v^32 + 14515200*u^23*v^31 - 120684288*u^23*v^30 + 144292224*u^23*v^29 - 145515200*u^23*v^32 + 14515200*u^23*v^31 - 120684288*u^23*v^30 + 144292224*u^23*v^29 - 145515200*u^23*v^33 + 3741696*u^23*v^31 - 120684288*u^23*v^30 + 144292224*u^23*v^29 - 145515200*u^23*v^33 + 3741696*u^23*v^31 - 120684288*u^23*v^30 + 144292224*u^23*v^29 - 145515200*u^23*v^33 + 3741696*u^23*v^33 + 3760*u^23*v^33 + 3760*u^ u^23*v^25 + 288648453912*u^23*v^24 + 409285388008*u^23*v^23 + 20427807704*u^23*v^22 - 1075680726296*u^23*v^21 - 2326037465032*u^23*v^20 - 2613175268904*u^23*v^19 $93214341616*u^{2}3*v^{1}3 - 250795805776*u^{2}3*v^{1}2 - 252189213488*u^{2}3*v^{1}1 - 123148371792*u^{2}3*v^{1}0 - 35656003376*u^{2}3*v^{9} - 4888395520*u^{2}3*v^{8} + 612248080*u^{2}3*v^{7} + 471675072*u^{2}3*v^{6} + 109150560*u^{2}3*v^{5}$ · 13271040*u^22*v^33 – 131983488*u^22*v^32 + 14129664*u^22*v^31 – 1744481536*u^22*v^30 + 2002814848*u^22*v ↓ + 9232141656*u^22*v^8 + 1413564832*u^22*v^7 + 131694736*u^22*v^6 + 6340032*u^22*v′ - 150027159392*u^21*v^29 - 711727075296*u^21*v^28 - 1352443802880*u^21*v^27 - 403107683480*u^21*v^20 20272100506608*u^21*v^18 - 6864189459600*u^21*v^17 + 2625874146528*u^21*v^16 + 5239931268384*u^21*v^15 + 3686226163312*u^21*v^14 + 1541521581344*u^21*v^13 + 351274029472*u^21*v^12 - 10079738544*u^2 7 + 3404704342516*u^20*v^26 - 15060414790024*u^20*v^25 - 40248633513442*u^20*v^24 - 52278208101600*u^20*v^23 - 35049696856160*u^20*v^22 +5146181339648*u^20*v^21 + 42352854221792*u^20*v^20 + xu^20*v^16 - 3238102766112*u^20*v^15 - 2982698910560*u^20*v^14 - 1399536207040*u^20*v^13 - 411445108944*u^20*v^12 - 66957467248*u^20*v^11 + 1257954152*u^20*v^10 + 3957240176*u^20*v^1 1276176384*u^19*v^36 - 4068430848*u^19*v^35 - 4429950720*u^19*v^34 + 139549441600*u^19*v^33 + 810050806656*u^19*v^32 + 1658835796672*u^19*v^31 - 6290277984*u^19*v^30 - $\frac{68705152 \times u^{19} \times v^{25}}{105425045321088 \times u^{19} \times v^{24}} + 136662145411984 \times u^{19} \times v^{23} + 101335863949024 \times u^{19} \times v^{22} + 22659412514992 \times u^{19} \times v^{22} + 22659412514992 \times u^{19} \times v^{20} - 70421435355744 \times u^{19} \times v^{20} + 22659412514992 \times u^{19} \times u^{19} \times v^{20} + 22659412514992 \times u^{19} \times v^{20} + 2265941251492 \times u^{19} \times v^{20} + 2265941251492 \times u^{19} \times v^{19} \times v^{19}$ <u>32576*u^19*v^14 + 763365</u>228688*u^19*v^13 + 252924701088*u^19*v^12 + 54738590352*u^19*v^11 + 7232189664*u^19*v^10 + 321322688*u^19*v^9 - 71350848*u^19*v^8 - 15152112*u^19*v^7 - 1329984*u^19*v^6 *v^35 - 107506546816*u^18*v^34 - 1723514593088*u^18*v^33 - 5847882189152*u^18*v^32 - 9193577992512*u^18*v^31 + 780955336720*u^18*v^30 + 38737356283384*u^18*v^29 + 97586078297724*u^18*v^28 + 4 - 256371453188736*u^18*v^23 - 199861893910592*u^18*v^22 - 79144514949024*u^18*v^21+ 21688714761280*u^18*v^20 + 60592857940416*u^18*v^19 + 511282578500 161362056*u^18*v^12 - 20212696896*u^18*v^11 - 3182195016*u^18*v^10 - 331212144*u^18*v^9 - 21059640*u^18*v^8 - 553824*u^18*v^7 + 13104*u^18*v^6 - 9823150 424958356768*u^17*v^33 + 29581202434528*u^17*v^32 + 39165957541696*u^17*v^31 + 1127136161224*u^17*v^29 - 263106482774560*u^17*v^28 - 339586632716912*u^17*v^27 - 244090667630016*u^17*v^26 274971143532176*u^17*v^22 + 131814145375696*u^17*v^21 + 16659203953824*u^17*v^20- 31381988144176*u^17*v^19 - 31840118172448*u^17*v^18 - 17575923020720*u^17*v^17- 6559406285632*u^17*v^16 - 1658822145920*u^17*v^15 -200*u^17*v^11 + 625706496*u^17*v^10 + 63593136*u^17*v^9 + 3732768*u^17*v^7 + 1423226880*u^16*v^39 + 144336393792*u^16*v^38 + 806087753856*u^16*v^37 + 2258374013120*u^16*v^36 + 1592182432 $109041767670650*u^{16*v^32} - 130688676228424*u^{16*v^31} - 29093518300868*u^{16*v^20} + 224029166141832*u^{16*v^29} + 516589295632790*u^{16*v^28} + 644634776345064*u^{16*v^27} + 487936614212828*u^{16*v^26} + 130688676228424*u^{16*v^27} + 487936614212828*u^{16*v^26} + 130688676228424*u^{16*v^27} + 487936614212828*u^{16*v^26} + 130688676228424*u^{16*v^27} + 130688676228424*u^{16*v^27} + 10008688*u^{16*v^27} + 1000868*u^{16*v^27} + 1000$ *u^16*v^22 - 138241746876592*u^16*v^21 - 38071283907988*u^16*v^20 + 5841070920048*u^16*v^19 +12729046981464*u^16*v^18 + 7631864008080*u^16*v^17 + 2911015365765*u^16*v^16 + 776244682152*u^16*v^15 3724*u^16*v^10 - 4662648*u^16*v^9 - 148959*u^16*v^8 - 1382658048*u^15*v^41 - 18200383488*u^15*v^39 - 1450167252480*u^15*v^38 - 5336495778560*u^15*v^37 - 10676003621376*u^15*v^36 - 4272192765696*u^1 487784*u^15*v^32+ 337080422305912*u^15*v^31 + 136692461804968*u^15*v^30 - 293093906240632*u^15*v^29 - 730590650259496*u^15*v^28 - 895216645564936*u^15*v^27 - 691540079708776*u^15*v^26 - 285911457280568*u^15*v^25 · 97511875665472*u^15*v^21 + 32944403259040*u^15*v^20 + 3996542316256*u^15*v^19 - 2784155804832*u^15*v^18 - 2149405860016*u^15*v^17 - 839790034408*u^15*v^16 - 220824889112*u^15*v^15 - 40600857896*u^15*v^14 -+ 422148096*u^14*v^42 + 9624305664*u^14*v^41 + 226060906752*u^14*v^40 + 1915168069632*u^14*v^39 + 8602918844928*u^14*v^38 + 23745413225728*u^14*v^37 + 3 4*v^21- 17065294088560*u^14*v^20 - 3577420083936*u^14*v^19 + 51501282536*u^14*v^18 + 362581809784*u^14*v^17 + 154635965516*u^14*v^16 44*u^13*v^42 - 130552579584*u^13*v^41 - 1804608287232*u^13*v^40 - 9995402193152*u^13*v^39 - 33924133347232*u^13*v^38 - 76914768026 784616*u^12*v^30 - 180188248264864*u^12*v^29 + 191733598693282*u^12*v^28 + 316462907672880*u^12*v^27 + 252348712438776*u^12*v^26 + 131608815655056*u^12*v^25 302993134072*u^12*v^19 - 43746230668*u^12*v^18 - 1133132360*u^12*v^17 + 1070329374*u^12*v^16 + 265999536*u^12*v^15 3441834437248×u^11×v^40 – 79074698373184×u^11×v^39 – 189267304555296×u^11×v^38 – 333865998012320×u^11×v^37 – 424932932532536×u^11×v^36 – 339169351427320×u^11×v^35 40560690160*u^10*v^29 - 26192445259080*u^10*v^28+ 15404759058112*u^10*v^27 + 18626542319992*u^10*v^26 + 10341545258320*u^10* 501984*u^10*v^17 + 91440*u^10*v^16 + 159120*u^10*v^15 + 8748*u^10*v^14 – 10100736*u^9*v^47 + 449948160 $70295646*u^{8*v^{36} + 269237879041848*u^{8*v^{35} + 173313089216820*u^{8*v^{34} + 72562749074744*u^{8*v^{33} + 5137895011307*u^{8*v^{33} - 20039371629520*u^{8*v^{26} - 4018423212152*u^{8*v^{27} + 23706422144*u^{8*v^{32} + 72562749074744*u^{8*v^{33} + 5137895011307*u^{8*v^{30} - 11048101276416*u^{8*v^{30} - 11048101276416*u^{8} - 11048101276416*u^{8}v^{30} - 1$ + 39276527180*u^8*v^24 + 9464244696*u^8*v^23 + 1399039388*u^8*v^22 + 97632616*u^8*v^21 - 5433066*u^8*v^20 - 1874952*u^8*v^16 - 2162276352*u^7*v^49 - 25794201600*u^7*v^48 - 145521054720*u^7*v^47 - 511932660480*u^7*v^46 - 1232760325760*u^7*v^45 - 1232760*u^7*v^45 - 1232760*u^7*v^7*v^45 - 1232760*u^7*v^ $\frac{1}{2} 299565684432684v^7kv^44 - 2636798584352ku^7kv^43 - 3448583387044v^7kv^42 - 8339347495764v^7kv^44 - 24765193681644v^7kv^44 - 55330985964v^7kv^37 - 124966551912ku^7kv^36 - 31262566004v^7kv^37 - 13466855884v^7kv^42 + 34313612632ku^7kv^38 - 132255084v^7kv^41 + 13283126328ku^7kv^42 + 343167675ku^7kv^7k^2 - 3419966528v^7kv^44 + 559774v^25 - 34209666524v^7kv^44 + 559774v^25 - 32603684v^7kv^44 + 559774v^25 - 326035884v^7kv^44 + 559774v^25 - 326035884v^7kv^44 + 559774v^25 - 326035884v^7kv^44 + 559774v^25 - 34208600k^7kv^42 + 586646158232ku^6kv^44 + 559784ku^6kv^34 + 555509835664v^6kv^44 + 559784v^25 - 4778788ku^6kv^24 + 4731392xu^6kv^23 + 190655884v^5kv^44 + 555509835664v^5kv^44 + 559784v^25 - 4778788ku^6kv^24 + 4731392xu^6kv^22 + 61512u^6kv^24 + 2738556416kv^5kv^24 - 3785556164kv^5kv^25 - 3907833948ku^6kv^34 - 655550832568v^5kv^44 - 63310421184v^5kv^44 + 63310421184v^5kv^44 + 63310421184v^5kv^44 - 6331042184v^5kv^44 - 632182864kv^4kv^44 - 888198888ku^4kv^44 - 881988684$ 2095650840320*u^7*v^44 – 2636798580352*u^7*v^43 – 3448583308704*u^7*v^42 – 8339348740576*u^7*v^41

